Simulating Black-Hole Radiation

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Abstract. Aspects of black-hole thermodynamics and the detectable consequences thereof are reviewed. A summary of current work into building a catalogue of black-hole emission spectra is given. Thereafter, the usefulness of such a catalogue in identifying decay processes of microscopic black-holes produced at modern particle colliders is discussed.

1. Introduction

That black-holes evaporate via the emission of thermal radiation is surprising discovery made by Hawking [1] and independently by Bekenstein [2]. Black-hole radiation combines the theories of general relativity and quantum mechanics to produce thermodynamics. As such, the black-hole is to a theory of quantum gravity what the hydrogen atom is to the theory of quantum mechanics.

Classical black-holes are accurately described by three parameters, namely their rest mass, angular momentum and charge. The thermodynamic properties of black-holes have also, to some degree of accuracy, been described in terms of these three parameters. This has lead to the a thermodynamic description of black-holes that was formalised in the four laws of black-hole mechanics as an analogue of the four classical laws of thermodynamics [3]. However, the microscopic degrees of freedom that lead to macroscopically thermal behaviour have yet to be identified.

Recent work has lead to an intriguing possibility of using classical, macroscopic properties of black-holes to infer information about their quantum, microscopic nature [4]. Quantum fluctuations at the event-horizon introduce particles into the region outside the black-hole horizon. The space-time around the black-hole is a potential barrier for the particles near the horizon. Particles that escape the potential are seen as radiation. This radiation is commonly referred to as *Hawking* radiation. Space-time itself is now the potential $V(\vec{x})$ to which quantum fields couple. Hawking radiation must tunnel through the potential barrier introduced by the black-hole.

There is a non-trivial coupling between the angular momentum of the black-hole and the spin of the emitted particles. This non-trivial coupling results in an enhancement of certain emission modes, and suppression of others, at different emission energies. Studies of these effects are governed by the appearance of Quasi-Normal Modes (QNM's) as the dominant modes of emission. QNM's also play an important role in black-hole stability studies [5]. QNM oscillations have been found in perturbation calculations of particles falling into Schwarzschild [6] and Kerr black-holes [7] and in the collapse of a star to form a black hole [8, 9, 10].

A full analytical treatment of the radiation emitted by black-holes parameterised by the black-hole rest mass and angular momentum, and emission energy is currently lacking in the literature. Moreover, an analytical description the angular distribution of the energy and angular momentum flux is lacking. A discussion of the formulation of black-hole radiation is presented below.

2. Geometric aspects of a Black-hole

Consider the metric for a $(D+1)$-dimensional black-hole with one rotation parameter. The line element in this black-hole space-time is

$$\begin{aligned} \mathrm{d}s^2 = &\left[1 - \frac{\mu r^{4-D}}{\Sigma(r,a,\theta)}\right]\mathrm{d}t^2 - \sin^2(\theta)\left[r^2 + a^2\sin^2(\theta)\frac{\mu r^{4-D}}{\Sigma(r,a,\theta)}\right]\mathrm{d}\phi^2 \\ &+ 2a\sin^2(\theta)\frac{\mu r^{4-D}}{\Sigma(r,a,\theta)}\mathrm{d}t\,\mathrm{d}\phi - \frac{\Sigma(r,a,\theta)}{\Delta(r,a,\mu,D)}\mathrm{d}r^2 - \Sigma(r,a,\theta)\mathrm{d}\theta^2 - r^2\cos^2(\theta)\,\mathrm{d}\Omega^2_{D-3} \end{aligned} \tag{1}$$

where μ is a related to the mass M of the black-hole, a is the dimensionless black-hole rotation parameter, and

$$\begin{aligned} \Sigma(r,a,\theta) =& r^2 + a^2\cos^2(\theta)\,, \\ \Delta(r,a,\mu,D) =& r^2 + a^2 - \mu r^{4-D}. \end{aligned}$$

The parameter a will be of major importance in characterising the emission of particles by black-holes.

The mass of the black-hole determined by

$$M = \frac{(D-1)A_{D-1}}{16\pi G_D}\mu$$

where

$$G_D = \frac{(2\pi)^{D-4}}{4M_D^{D-1}}$$

is the higher dimensional gravitational constant and

$$A_{D-1} = \frac{2\pi^{D/2}}{\Gamma(D/2)}$$

is the hyper-surface area of a $(D-1)$-dimensional unit sphere.

The black-hole event horizon is specified by the assignment of $\Delta(r,a,\mu,D) = 0$, where the radius of the a event horizon given implicitly by

$$r_h^{(D)} = \left[\frac{\mu}{1 - \left(\frac{a}{r_h^{(D)}}\right)^2}\right]^{\frac{1}{D-2}}.$$

The Schwarzschild radius of a $(D+1)$-dimensional black-hole is $r_s^{(D)} = \mu^{\frac{1}{D-2}}$, in which case,

$$r_h^{(D)} = r_s^{(D)}\left[1 - \left(\frac{a}{r_h^{(D)}}\right)^2\right]^{\frac{-1}{D-2}}. \tag{2}$$

Jacobson [4] showed that the Einstein Equation should be thought of as an equation of state in thermodynamic system. The equation of state is resolved in the thermodynamic limit as a relationship between thermodynamic variables. Following this, it is reasonable that one should want to supply a description of the micro-states of space-time in some natural way. The entropy relationship of a black-hole is a natural realisation of this statement where

$$T_H \delta S = \delta M - \sum_{i=1}^{N} \Omega_i \delta J_i, \tag{3}$$

where T_H is the *Hawking temperature* of the black-hole, S and M are the entropy and mass of the black-hole, and J_i and Ω_i are the charges and their conjugate chemical potentials [3]. This is analogous to the well known equation of state for a system in classical thermodynamics,

$$T\delta S = \delta Q, \tag{4}$$

where the classical heat of the system Q is connected to the entropy of the system S at temperature T. After solving (2) implicitly for $r_h^{(D)}$, it can be shown that T_H is given by

$$T_H = \frac{D-2}{4\pi r_h^{(D)}}. \tag{5}$$

It may be shown that S is proportional to the area of the black-hole horizon. The Hawking temperature T_H is then identically the classical temperature of the system T. The next section discusses how the emission of Hawking radiation by a black-hole is analogous to the emission of thermal radiation by a black-body radiator.

3. Black-Hole Radiation

Since quanta of each field in the theory can be emitted by the black-hole, each choice of matter content in the theory will effect the final emission spectrum of the black-hole. A generic choice of matter fields may be included in the action of the classical black-hole-quantum field system,

$$S = \int_{\partial M} \mathrm{d}^N x \sqrt{\det(g_M)} \left(R + g_M^{\mu\nu} \partial_\mu \phi_i \partial_\nu \phi_i^\dagger + m^2 \phi_i \phi_i^\dagger + \dots \right), \tag{6}$$

where g_M is the space-time metric for a specific black-hole. Extremising (6) yields the generalised Klein-Gordon equation,

$$\left(\hat{\Delta} + k^2 \right) \phi_i = 0, \tag{7}$$

where where $\hat{\Delta}$ is the *Laplace-Beltrami* operator defined by the unperturbed metric g_M and $k \in \mathbb{C}$. The eigenvalue problem of (7) is then of principle interest since the total flux of each field ϕ_i is weighted by the probability density $|\phi_i|^2$. Solving (7) constitutes solving the equation of motion of a given field in the region of a potential.

Hawking used a semi-classical approximation to show that black holes have an exact thermal spectrum, where the expectation value $\langle n_i(\omega) \rangle$ for the number of particles of the i-th species, emitted in a mode with frequency ω, is given by

$$\langle n_i(\omega) \rangle = \frac{\gamma_i(\omega)}{e^{\frac{\omega}{T_H}} \pm 1} \tag{8}$$

where the plus sign describes fermions and the minus sign describe bosons and where $\gamma_i(\omega)$ is the transmission or *grey-body* factor which is the probability for an outgoing (incoming) wave, in the ω-mode, to reach infinity. It is this grey-body factor that characterises the radiation of ϕ_i by a given black-hole, and is dependent on the spin of that field and the rotation parameter a.

It should be noted that the semi-classical calculation of Hawking emission is only reliable when the energy of the emitted particle is small compared to the black hole mass $\omega \ll M$, since only in this case is it correct to neglect the back reaction of the metric during the emission process.

The power emission spectrum for ϕ_i can be computed using

$$\frac{\mathrm{d}^2 E^{(i)}}{\mathrm{d}t\mathrm{d}\omega} = \sum_{l,m} \frac{\omega}{e^{\frac{\omega}{T_H}} \pm 1} \frac{N^{(i)}_{l,m} \left| A^{(i)}_{l,m} \right|^2}{2\pi}, \tag{9}$$

where $N^{(i)}_{l,m}$ and $\left| A^{(i)}_{l,m} \right|^2$ are the (l,m)-state degeneracy factor and the outgoing particle probability density ϕ_i. The $\frac{1}{e^{\omega/T_H} \pm 1}$ factor in (9) is reminiscent of the statistical distribution of the black-body spectrum of a classical black-body radiator. The contribution to the emission spectrum by field ϕ_i has a distribution similar to a black-body radiator that peaks at the QNM frequency for that field. Integrating (8), or equivalently (9), over all particle types i yields the total emissivity emission power spectrum for given black-hole in a theory with a given matter content. Lacking analytical solutions to the emission power spectra, these integrals are computed numerically.

4. Simulated Black-Hole Production at Particle Colliders

Recent interest in the Hawking radiation and QNMs of black-holes has stemmed from the idea that mini-TeV scale black-holes might be created in particle accelerators such as the Large Hadron Collider (LHC) [11, 12, 13, 14, 15]. Once the radiation emission signatures for these mini-black-holes is characterised one should be able to either confirm or rule out TeV-scale black-hole production. More generally, it is expected that once the emission signatures of black-hole radiation are completely known, some useful insight into the possible limits on the micro-states of black-holes, and hence space-time will be gained.

Grey-body factors are important experimentally because they modify the spectrum in the region where most particles are produced thus, altering the characteristic spectrum by which a 'black-hole event' would be identified. Hence, each black-hole decay is characterised by the collection of grey-body factors defined by the given theory where each stage of black-hole decay in a particle collider traces a trajectory through the parameter space of grey-body factors and black-hole states.

There has been extensive work on a collection of black-hole event generators. Work is currently underway on a new version of an event generator called *BlackMax*. BlackMax is a testbed for simulated black-hole events which generates particle emission spectra. There is a wide range a possible customisations that can be made to the generator at run-time. BlackMax is designed to accommodate additional black-hole geometries and decay processes without modification. BlackMax uses numerically computed power spectra to produce the correctly weighted particle emission spectra. The goal of the BlackMax project is to generate black-hole decay signatures for use in data analysis at LHC. According to the expected detector performance specifications, should TeV scale black-holes be produced within LHC, there is a high probability of discovery [16, 17]. The goal of the BlackMax project is to produce black-hole decay signatures which would serve as templates for the actual black-hole decay. Given a complete catalogue of black-hole decay signatures, there is opportunity to either verify or eliminate of TeV gravity models at modern particle detectors.

5. Summary and Outlook

The exact nature of black-hole micro-states, and hence the micro-states of space-time, is unknown, but by studying the thermodynamic properties of black-holes it is hoped the some insight may be gained into the nature of these micro-states. A significant open question to the nature of black-hole thermodynamics is to determine the analytical solution to the black-hole power spectrum. Also, black-hole radiation is a function of the grey-body factors $\gamma_i(\omega)$ it is suggested that the emission of particles by black-holes might not be thermal [18, 19], which would have consequences for the information loss problem for black-holes. From this work and work to come it is hoped that black-hole micro-states, or at least hints to what these micro-states might be, can be reverse engineered from the integrated power spectra of this and future work. In addition, black-hole decay signature cataloguing is expected to either validate or invalidate theories of low scale quantum gravity.

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